8.6 Factor $ax^2 + bx + c$

Before	You factored trinomials of the form $x^2 + bx + c$.
Now	You will factor trinomials of the form $ax^2 + bx + c$.
Why?	So you can find the dimensions of a building, as in Ex. 61.

Key Vocabulary
• trinomial

COMMON CORE

CC.9-12.A.SSE.3a Factor a quadratic expression to reveal the zeros of the function it defines.

REVIEW FACTORING
You may want to review determining the signs of the factors of a trinomial.

When factoring a trinomial of the form $ax^2 + bx + c$, first consider the signs of b and c, as shown below. This approach works when a is positive.

EXAMPLE 1 Factor when *b* is negative and *c* is positive

Factor $2x^2 - 7x + 3$.

Solution

Because b is negative and c is positive, both factors of c must be negative. Make a table to organize your work.

You must consider the order of the factors of 3, because the x-terms of the possible factorizations are different.

Factors of 2	Factors of 3	Possible factorization	Middle term when multiplied	
1, 2	−1, −3	$(x - 1)(2x - 3)$	$-3x - 2x = -5x$	✗
1, 2	**−3, −1**	$(\mathbf{x} - \mathbf{3})(\mathbf{2x} - \mathbf{1})$	$-x - 6x = -7x$	← Correct

▶ $2x^2 - 7x + 3 = (x - 3)(2x - 1)$

EXAMPLE 2 Factor when *b* is positive and *c* is negative

Factor $3n^2 + 14n - 5$.

Solution

Because b is positive and c is negative, the factors of c have different signs.

Factors of 3	Factors of −5	Possible factorization	Middle term when multiplied	
1, 3	1, −5	$(n + 1)(3n - 5)$	$-5n + 3n = -2n$	✗
1, 3	−1, 5	$(n - 1)(3n + 5)$	$5n - 3n = 2n$	✗
1, 3	**5, −1**	$(\mathbf{n} + \mathbf{5})(\mathbf{3n} - \mathbf{1})$	$-n + 15n = 14n$	← Correct
1, 3	−5, 1	$(n - 5)(3n + 1)$	$n - 15n = -14n$	✗

▶ $3n^2 + 14n - 5 = (n + 5)(3n - 1)$

GUIDED PRACTICE for Examples 1 and 2

Factor the trinomial.

1. $3t^2 + 8t + 4$

2. $4s^2 - 9s + 5$

3. $2h^2 + 13h - 7$

FACTORING WHEN a IS NEGATIVE To factor a trinomial of the form $ax^2 + bx + c$ when a is negative, first factor -1 from each term of the trinomial. Then factor the resulting trinomial as in the previous examples.

❖ **EXAMPLE 3** **Factor when a is negative**

Factor $-4x^2 + 12x + 7$.

Solution

STEP 1 **Factor** -1 from each term of the trinomial.

$$-4x^2 + 12x + 7 = -(4x^2 - 12x - 7)$$

STEP 2 **Factor** the trinomial $4x^2 - 12x - 7$. Because b and c are both negative, the factors of c must have different signs. As in the previous examples, use a table to organize information about the factors of a and c.

| Factors of 4	Factors of -7	Possible factorization	Middle term when multiplied	
1, 4	1, -7	$(x + 1)(4x - 7)$	$-7x + 4x = -3x$	✗
1, 4	7, -1	$(x + 7)(4x - 1)$	$-x + 28x = 27x$	✗
1, 4	-1, 7	$(x - 1)(4x + 7)$	$7x - 4x = 3x$	✗
1, 4	-7, 1	$(x - 7)(4x + 1)$	$x - 28x = -27x$	✗
2, 2	**1, -7**	**$(2x + 1)(2x - 7)$**	$-14x + 2x = -12x$	← Correct
2, 2	-1, 7	$(2x - 1)(2x + 7)$	$14x - 2x = 12x$	✗

AVOID ERRORS
Remember to include the -1 that you factored out in Step 1.

▶ $-4x^2 + 12x + 7 = -(2x + 1)(2x - 7)$

CHECK You can check your factorization using a graphing calculator. Graph $y_1 = -4x^2 + 12x + 7$ and $y_2 = -(2x + 1)(2x - 7)$. Because the graphs coincide, you know that your factorization is correct.
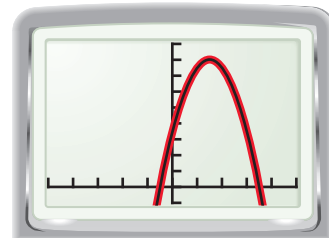
GUIDED PRACTICE for Example 3

Factor the trinomial.

4. $-2y^2 - 5y - 3$

5. $-5m^2 + 6m - 1$

6. $-3x^2 - x + 2$

FINDING A COMMON FACTOR You have learned to factor out the greatest common monomial factor from the terms of a polynomial. Sometimes you may need to do this before finding two binomial factors of a trinomial.

EXAMPLE 4 **Write and solve a polynomial equation**

DISCUS An athlete throws a discus from an initial height of 6 feet and with an initial vertical velocity of 46 feet per second.

a. Write an equation that gives the height (in feet) of the discus as a function of the time (in seconds) since it left the athlete's hand.

b. After how many seconds does the discus hit the ground?

Solution

a. Use the vertical motion model to write an equation for the height h (in feet) of the discus. In this case, $v = 46$ and $s = 6$.

$h = -16t^2 + \boldsymbol{v}t + \boldsymbol{s}$ **Vertical motion model**

$h = -16t^2 + \boldsymbol{46}t + \boldsymbol{6}$ **Substitute 46 for v and 6 for s.**

b. To find the number of seconds that pass before the discus lands, find the value of t for which the height of the discus is 0. Substitute 0 for h and solve the equation for t.

$0 = -16t^2 + 46t + 6$ **Substitute 0 for h.**

$0 = -2(8t^2 - 23t - 3)$ **Factor out -2.**

$0 = -2(8t + 1)(t - 3)$ **Factor the trinomial. Find factors of 8 and -3 that produce a middle term with a coefficient of -23.**

$8t + 1 = 0$ *or* $t - 3 = 0$ **Zero-product property**

$t = -\dfrac{1}{8}$ *or* $t = 3$ **Solve for t.**

The solutions of the equation are $-\dfrac{1}{8}$ and 3. A negative solution does not make sense in this situation, so disregard $-\dfrac{1}{8}$.

▶ The discus hits the ground after 3 seconds.

✔ **GUIDED PRACTICE**   **for Example 4**

7. WHAT IF? In Example 4, suppose another athlete throws the discus with an initial vertical velocity of 38 feet per second and releases it from a height of 5 feet. After how many seconds does the discus hit the ground?

8. SHOT PUT In a shot put event, an athlete throws the shot put from an initial height of 6 feet and with an initial vertical velocity of 29 feet per second. After how many seconds does the shot put hit the ground?

A rectangle's length is 13 meters more than 3 times its width. The area is 10 square meters. What is the width?

Ⓐ $\frac{2}{3}$ m   Ⓑ 3 m   Ⓒ 5 m   Ⓓ 10 m

$w(3w + 13) = 10$	**Write an equation to model area.**
$3w^2 + 13w - 10 = 0$	**Simplify and subtract 10 from each side.**
$(w + 5)(3w - 2) = 0$	**Factor left side.**
$w + 5 = 0$ or $3w - 2 = 0$	**Zero-product property**
$w = -5$ or $w = \frac{2}{3}$	**Solve for *w*.**

Reject the negative width.

▶ The correct answer is A. Ⓐ Ⓑ Ⓒ Ⓓ

✓ **GUIDED PRACTICE**   for Example 5

9. A rectangle's length is 1 inch more than twice its width. The area is 6 square inches. What is the width?

   Ⓐ $\frac{1}{2}$ in.   Ⓑ $\frac{3}{2}$ in.   Ⓒ 2 in.   Ⓓ $\frac{5}{2}$ in.

8.6 EXERCISES

SKILL PRACTICE

1. **VOCABULARY** What is another word for the solutions of $x^2 + 2x + 1 = 0$?

2. ★ **WRITING** *Explain* how you can use a graph to check a factorization.

3. ★ **WRITING** *Compare* factoring $6x^2 - x - 2$ with factoring $x^2 - x - 2$.

EXAMPLES 1, 2, and 3
for Exs. 4–22

FACTORING TRINOMIALS Factor the trinomial.

4. $-x^2 + x + 20$ 5. $-y^2 + 2y + 8$ 6. $-a^2 + 12a - 27$

7. $5w^2 - 6w + 1$ 8. $-3p^2 - 10p - 3$ 9. $6s^2 - s - 5$

10. $2t^2 + 5t - 63$ 11. $2c^2 - 7c + 3$ 12. $3n^2 - 17n + 10$

13. $-2h^2 + 5h + 3$ 14. $-6k^2 - 13k - 6$ 15. $10x^2 - 3x - 27$

16. $4m^2 + 9m + 5$ 17. $3z^2 + z - 14$ 18. $4a^2 + 9a - 9$

19. $4n^2 + 16n + 15$ 20. $-5b^2 + 7b - 2$ 21. $6y^2 - 5y - 4$

22. ★ **MULTIPLE CHOICE** What is the correct factorization of $8x^2 - 10x + 3$?

    A $(2x - 3)(4x - 1)$ **B** $(2x - 1)(4x - 3)$

    C $(4x + 1)(2x - 3)$ **D** $(8x - 3)(x - 1)$

SOLVING EQUATIONS Solve the equation.

23. $2x^2 - 3x - 35 = 0$ **24.** $3w^2 + 22w + 7 = 0$ **25.** $4s^2 + 11s - 3 = 0$

26. $7a^2 + 2a = 5$ **27.** $8t^2 - 2t = 3$ **28.** $6m^2 - 5m = 14$

29. $b(20b - 3) - 2 = 0$ **30.** $4(3y^2 - 7y + 4) = 1$ **31.** $p(3p + 14) = 5$

32. $4n^2 - 2n - 90 = 0$ **33.** $10c^2 - 14c + 4 = 0$ **34.** $-16k^2 + 8k + 24 = 0$

35. $6r^2 - 15r = 99$ **36.** $56z^2 + 2 = 22z$ **37.** $30x^2 + 25x = 20$

ERROR ANALYSIS *Describe* and correct the error in solving the equation.

38.

$5x^2 + x = 4$

$x(5x + 1) = 4$

$x = 4 \text{ or } 5x + 1 = 4$

$x = 4 \text{ or } \quad x = \dfrac{3}{5}$

39.

$12x^2 + 5x - 2 = 0$

$(3x - 1)(4x + 2) = 0$

$3x - 1 = 0 \text{ or } 4x + 2 = 0$

$x = \dfrac{1}{3} \text{ or } \quad x = -\dfrac{1}{2}$

40. **GEOMETRY** The length of a rectangle is 7 inches more than 5 times its width. The area of the rectangle is 6 square inches. What is the width?

41. ★ **SHORT RESPONSE** The length of a rectangle is 1 inch more than 4 times its width. The area of the rectangle is 3 square inches. What is the perimeter of the rectangle? *Explain* how you found your answer.

FINDING ZEROS OF FUNCTIONS Find the zeros of the polynomial function.

42. $g(x) = 2x^2 + x - 1$ **43.** $f(x) = -x^2 + 12x - 35$ **44.** $h(x) = -3x^2 + 2x + 5$

45. $f(x) = 3x^2 + x - 14$ **46.** $g(x) = 8x^2 - 6x - 14$ **47.** $f(x) = 12x^2 - 24x - 63$

SOLVING EQUATIONS Multiply each side of the equation by an appropriate power of 10 to obtain integer coefficients. Then solve the equation.

48. $0.3x^2 - 0.7x - 4.0 = 0$ **49.** $0.8x^2 - 1.8x - 0.5 = 0$ **50.** $0.4x^2 - 0.4x = 9.9$

51. ★ **MULTIPLE CHOICE** What are the solutions of the equation $0.4x^2 - 1.1x = 2$?

    A -12.5 and 40 **B** -4 and 1.25 **C** -1.25 and 4 **D** -0.125 and 0.4

WRITING EQUATIONS Write a polynomial equation that has the given solutions. The equation must have integer coefficients. *Explain* your reasoning.

52. -3 and 2 **53.** $-\dfrac{1}{2}$ and 5 **54.** $-\dfrac{3}{4}$ and $-\dfrac{1}{3}$

CHALLENGE Factor the trinomial.

55. $2x^2 - 11xy + 5y^2$ **56.** $3x^2 + 2xy - 8y^2$ **57.** $6x^3 - 10x^2y - 56xy^2$

58. DIVING A diver dives from a cliff when her center of gravity is 46 feet above the surface of the water. Her initial vertical velocity leaving the cliff is 9 feet per second. After how many seconds does her center of gravity enter the water?

59. SCRAPBOOK DESIGN You plan to make a scrapbook. On the cover, you want to show three pictures with space between them, as shown. Each of the pictures is twice as long as it is wide.
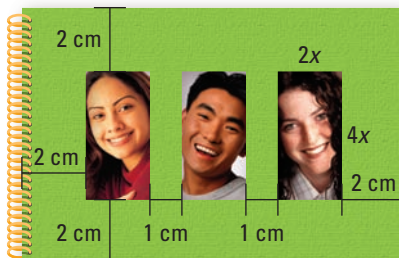
a. Write a polynomial that represents the area of the scrapbook cover.

b. The area of the cover will be 96 square centimeters. Find the length and width of the pictures you will use.

2 cm

2 cm

2 cm

2 cm   1 cm   1 cm

2x

4x

2 cm

60. ★ SHORT RESPONSE You throw a ball into the air with an initial vertical velocity of 31 feet per second. The ball leaves your hand when it is 6 feet above the ground. You catch the ball when it reaches a height of 4 feet. After how many seconds do you catch the ball? *Explain* how you can use the solutions of an equation to find your answer.

61. PARTHENON The Parthenon in Athens, Greece, is an ancient structure that has a rectangular base. The length of the Parthenon's base is 8 meters more than twice its width. The area of the base is about 2170 square meters. Find the length and width of the Parthenon's base.

62. ◆◆ MULTIPLE REPRESENTATIONS An African cat called a serval leaps from the ground in an attempt to catch a bird. The serval's initial vertical velocity is 24 feet per second.
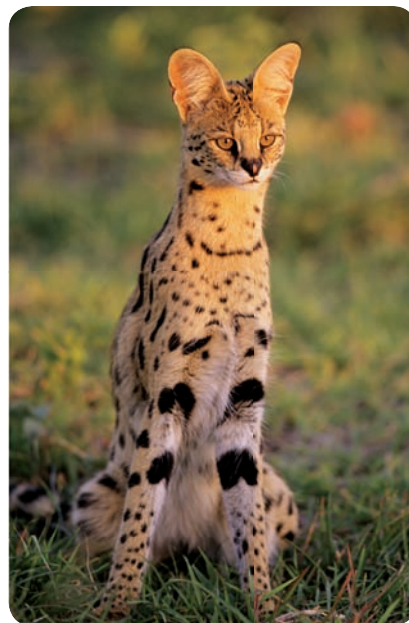
a. **Writing an Equation** Write an equation that gives the serval's height (in feet) as a function of the time (in seconds) since it left the ground.

b. **Making a Table** Use the equation from part (a) to make a table that shows the height of the serval for $t = 0, 0.3, 0.6, 0.9, 1.2,$ and 1.5 seconds.

c. **Drawing a Graph** Plot the ordered pairs in the table as points in a coordinate plane. Connect the points with a smooth curve. After how many seconds does the serval reach a height of 9 feet? *Justify* your answer using the equation from part (a).

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○ = See **WORKED-OUT SOLUTIONS** in Student Resources

★ = **STANDARDIZED TEST PRACTICE**

◆◆ = **MULTIPLE REPRESENTATIONS**

63. CHALLENGE A bush cricket jumps from the ground into the air with an initial vertical velocity of 4 feet per second.

  a. Write an equation that gives the cricket's height (in feet) as a function of the time (in seconds) since it left the ground.

  b. After how many seconds is the cricket 3 inches off the ground?

  c. Does the cricket jump higher than 3 inches? *Explain* your reasoning using your answer from part (b).

QUIZ

Factor out the greatest common monomial factor.

  1. $16a^2 - 40b$

  2. $9xy^2 + 6x^2y$

  3. $4n^4 - 22n^3 - 8n^2$

  4. $3x^2 + 6xy - 3y^2$

  5. $12abc^2 - 6a^2c$

  6. $-36s^3 + 18s^2 - 54s$

Factor the trinomial.

  7. $r^2 + 15r + 56$

  8. $s^2 - 6s + 5$

  9. $w^2 + 6w - 40$

  10. $-a^2 + 9a + 22$

  11. $2x^2 - 9x + 4$

  12. $5m^2 + m - 6$

  13. $6h^2 - 19h + 3$

  14. $-7y^2 - 23y - 6$

  15. $18c^2 + 12c - 6$

Solve the equation.

  16. $(4p - 7)(p + 5) = 0$

  17. $-8u^2 + 28u = 0$

  18. $51x^2 = -17x$

  19. $b^2 - 11b = -24$

  20. $m^2 + 12m = -35$

  21. $q^2 + 19 = -20q$

  22. $3t^2 - 11t + 10 = 0$

  23. $4y^2 + 31y = 8$

  24. $14s^2 + 12s = 2$

25. BASEBALL A baseball player hits a baseball into the air with an initial vertical velocity of 72 feet per second. The player hits the ball from a height of 3 feet.

  a. Write an equation that gives the baseball's height as a function of the time (in seconds) after it is hit.

  b. After how many seconds is the baseball 84 feet above the ground?